Stochastically bounded Solutions of stochastic integrodifferential equations modeling neural networks

Zhenkun Huang, Honghua Bin

Abstract— In these paper, by using stochastic integral properties about solutions of homogeneous linear equations and fixed-point theorem, we investigate bounded dynamics of neural networks with stochastic effects and distributed delays. Some new criteria for the existence of a unique stochastic bounded solution of stochastic newworks are given.

Keywords — Stochastically bounded; stochastic neural networks; stochastic integral; distributed delays.

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1 INTRODUCTION

Stochastic differential equations and relative applications have recently been studied intensively [1, 2, 3]. It is of great interest to discuss qualitative behavior such as stochastic boundedness, exponential stability, periodicity or almost periodicity [6, 7, 8, 11] and so on. It is well known that stochastic boundedness of stochastic differential equations depend on it's linearized homogeneous equations [9]. For this research direction, II'chenko [5] established existence of a unique stochastically bounded solution of a linear nonhomogeneous differential eqution. Later, Luo [10] extended relative results to a class nonlinear stochastic differential eqution and reported some criteria foe existence of a stochastically bounded solution. It is shown that such stochastically bounded solutions can inherit properties of the coefficients of the equation if they are either stationary or periodic.

Meanwhile, there will exist interest qualitative behavior about stochastic boundedness for neural networks with stochastic perturbation. However, so far little is known about the existence of a unique stochastically bounded solutions of nerual networks and the aim of this paper to is close this gap.

In the present paper, we consider the following stochastic neural networks with distributed delays [4]

$$
\begin{aligned}
dx_i(t) &= [-a_i x_i(t) + \sum_{j=1}^{M} b_{ij} f_j(x_j(t)) \\
&\quad + \sum_{j=1}^{M} \int_{-\infty}^{t} k_{ij}(t-u) g_j(x_j(u)) du + I_i] dt \qquad (1.1) \\
&\quad + \sum_{j=1}^{M} h_{ij}(x_j(t)) dw_j(t),
\end{aligned}
$$

Where $i \in \mathrm{M} := \{1, 2, \cdots, M\}, w(t) = (w_1(t), \cdots w_M(t))^T$ is M-dimensional independent Wiener processes with respect to a probability space $(\Omega, F, F_t, -\infty < t < \infty, P)$.Throughout this paper, for each $i, j \in \mathrm{M}$,we suppose some basic assumptions:

Assumption 1. $a_i > 0,\ b_{ij}$ and I_i are real constants; $\sup_{v\in\mathbb{R}} |f_j(v)| \le B_{f_j} < +\infty$, $\sup_{v\in\mathbb{R}} |g_j(v)| \le B_{g_j} < +\infty$, $\sup_{v\in\mathbb{R}} |h_{ij}^{\dagger}(v)| \le B_{h_{ij}^{\dagger}} < +\infty$; Moreover, $h_j(\cdot)$, $g_j(\cdot)$ and $h_{ij}^{\dagger}(\cdot)$ are Lipschitz-continuous with Lipschitz constant $L_{f_j} > 0,\ L_{g_j} > 0$ and $\mathrm{L}_{h_{ij}^{\dagger}} > 0,$ respectively. $h_{ij}^{\dagger}(x) := h_{ij}(x) - \alpha_{ij} x,$ where $\alpha_{ij} \ne 0$ if $i = j$ and $\alpha_{ij} = 0$ if $i \ne j$.

Assumption 2. Theconvolution-type kernel $k_{ij}(\cdot)$ is in $L^1(0, +\infty)$ and satisfy with $\int_0^{+\infty} k_{ij}(v) dv = k_{ij}^* \in (0, +\infty)$.

Based on some stochastic integral properties and fixed-point theorem, we establish new criteria for the existence of a unique stochastically bounded solution for (1.1) The nonautonomous cases are alse considered.

2 Main results

Let $(\mathbb{R}^M, \|\cdot\|)$ be a Banach space. The collection of all measurable, square-integrable random variables, denoted by $L^2(P, \mathbb{R}^M)$, equipped with norm $\| X \|_{L^2(P,\mathbb{R}^M)} = (E \| X \|^2)^{1/2}$, where and the expectation E is defined by $E[g] = \int_{\Omega} g(\omega) dP(\omega)$. Define $B_E(\mathbb{R}, L^2(P, \mathbb{R}^M))$ to be the collection of all stochastic process $x: \mathbb{R} \to L^2(P, \mathbb{R}^M)$, which are continuous and bounded in quadratic mean. It is then easy to check that $B_E(\mathbb{R}, L^2(P, \mathbb{R}^M))$ is a Banach space when it is equipped with the norm

$$\| X \|_{\infty} = \sup_{t\in\mathbb{R}} (E \| X \|^2)^{1/2}.$$

For any given $i \in \mathrm{M}$,It $\hat{o}$ -type homogeneous linear eqution [1]

$$dx_i(t) = -\alpha_i x_i(t) dt + \alpha_{ij} x_i(t) dw_i(t)$$

has a solution

$$\lambda_i^s(t) = \exp\{a_i^*(t-s) + \alpha_{ii}[\omega_i(t) - \omega_i(s)]\}$$

which satisfys with the initial condition $x_i(s)=1$, where $a_i^* := -a_i - 2^{-1}a_{ii}^2 < 0$. For any arbitrary $p \in \mathbb{R}$,

$$\mathrm{E}(\lambda_i^s(t))^p := \exp\{[a_i^* + p2^{-1}a_{ii}^2](t-s)p\}. \qquad (2.1)$$

By Itô's formula, we can check that $x^s(t)$ of (1.1) can be represented in the following form

$$\begin{aligned} x_i^s(t) &= \lambda_i^s(t)[x_i(s) + \int_s^t (\lambda_i^s(u))^{-1}[\sum_{j=1}^M b_{ij} f_j(x_j^s(u)) \\ &+ \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) g_j(x_j^s(v))dv + I_i - \alpha_{ii} h_{ii}^\dagger(x_i^s(u))]du \\ &+ \sum_{i=1}^M \int_s^t (\lambda_i^s(u))^{-1} h_{ij}^\dagger(x_j^s(u))dw_j(u)], \quad i \in \mathrm{M} \end{aligned} \qquad (2.2)$$

The following basic definition and three lemmas are essential in the proof of our main results.

Definition 2.1. A solution $x(t), t \in \mathbb{R}$, of (1.1) is said to be stochastically bounded if

$$\lim_{N\to+\infty} \sup_{t\in\mathbb{R}} P\{|x_i(t)| > N\} = 0$$

holds for each $i \in \mathrm{M}$.

Lemma 2.1. ([5]) Assume that $\psi(t)$ is a continuous bounded function on $t \in \mathbb{R}$. Then the following reverse integral formula holds for $s \le t$:

$$\lambda_i^s(t)\int_s^t (\lambda_i^s(u))^{-1} \psi(u)dw_i(u) = -\int_t^s \lambda_i^u(t)\psi(u)dw_i(u) - \alpha_{ii}\int_t^s \lambda_i^u(t)\psi(u)du,$$

where $i \in \mathrm{M}$.

Lemma 2.2. ([5]) Assume that $\psi(t)$ is a continuous function with $\sup_{t\in\mathbb{R}} |\psi(t)| \le K < +\infty$. For each $i \in \mathrm{M}$, one gets

(i) for $\forall r \in \mathbb{R}$ and $p \in (0, 1+2a_i\alpha_{ii}^{-2})$, there are constants $T>0, L=L(K)>0$ and $0<q<1$ such that

$$P\{\int_{t-T(n+1)}^{t-Tn} (\lambda_i^u(t)|\psi(u)|)^r dr > N^r 2^{-n}\} \le LN^{-P}q^n$$

for all $t \in \mathbb{R}$, $N>0$ and $n \in \mathbb{N}$;

(ii) the following two limits

$$\begin{cases} \lim_{s\to-\infty} \int_t^s \lambda_i^u(t)\psi(u)du = \int_t^{-\infty} \lambda_i^u(t)\psi(u)du \\ \lim_{s\to-\infty} \int_t^s \lambda_i^u(t)\psi(u)d\omega_i(u) = \int_t^{-\infty} \lambda_i^u(t)\psi(u)d\omega_i(u) \end{cases}$$

exist almost surely for all $t \in \mathbb{R}$, respectively.

Lemma 2.3. ([12]) Let $\{X_t, F_t; 0 \le t \le +\infty\}$ be a submartingale whose every path is right-continuous, let $[\sigma,\tau]$ be a suninterval of $[0,+\infty)$. Then Doob's maximal inequality holds:

$$E(\sup_{\sigma\le t\le\tau} X_t)^p \le (\frac{p}{p-1})E(X_r^p),\ p>1$$

Provided $X_t \ge 0$ a.s. P for every $t \ge 0$, and $E(X_r^p) < +\infty$.

For simplicity, for real constant c, denote $c^\hbar := |c|$. Then our main result follows as:

Theorem 2.1. There exists a unique stochastically bounded solution $x^{-\infty}(t), t \in \mathbb{R}$, of (1.1) if $2a_i > a_{ii}^2, i \in \mathrm{M}$, and

$$\Theta := \max_{i\in\mathrm{M}}\{3M\sum_{j=1}^M[\frac{(L_{f_j}b_{ij}^\hbar)^2}{a_i^* a_{i1}^{*\dagger\dagger\dagger}} + \frac{(L_{g_j})^2 k_{ij}^*}{a_i a_{i1}} + \frac{(L_{h_{ij}^\dagger})^2}{a_{i2}}]\} < 1,$$

Where $a_{i1}^{\dagger*} := \alpha_i^{\,1} + 2^{-}\alpha_{ii}$ and $a_{i2}^{\dagger*} := 2(\alpha_i + \alpha_{ii})$. In this case of the existence, we have

$$\begin{aligned} x^{-\infty}(t) := -\int_t^{-\infty} \lambda_i^u(t)[\sum_{j=1}^M b_{ij} f_j(x_j^{-\infty}(u)) \\ + \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) g_j(x_j^{-\infty}(u))dv + I_i]du \\ + \sum_{j=1}^M \int_t^{-\infty} \lambda_i^u(t) h_{ij}^\dagger(x^{-\infty}(u))d\omega_j(u), \quad i \in \mathrm{M}. \end{aligned} \qquad (2.3)$$

Moreover,

$$\sup_{t\in\mathbb{R}} E\|x^{-\infty}(t)\|^2 < +\infty.$$

Proof. By Lemma 2.1, we get from (2.2) that

$$\begin{aligned} x_i^s(t) &= \lambda_i^s(t)x_i(s) - \sum_{j=1}^M \int_t^s (\lambda_i^u(t))^{-1} h_{ij}^\dagger(x_j^s(u))d\omega_j(u) \\ &- \int_t^s (\lambda_i^u(t))^{-1}[b_{ij} f_j(x_j^s(u)) \\ &+ \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) g_j(x_j^s(v))dv + I_i]du \end{aligned}$$

Putting $x(s)=0$ in the above and approaching the limit as $s \to -\infty$, due to Lemma 2.2, we get the limit $x^{-\infty}(t)$ which is a solution of (1.1). The process $x^{-\infty}(t)$, $t \in \mathbb{R}$, is measurable with respect to the flow $\Gamma_t := \sigma\{\omega_k(s_2) - \omega_k(s_1) : s_1 \le s_2 \le t, k \in \mathrm{M}\}$.

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For any given $z^{-\infty}(t) \in B_E(\mathbb{R}, L^2(P, \mathbb{R}^M))$, define $\varphi(z^{-\infty}(t)) := (\varphi_1(z^{-\infty}(t)), \cdots, \varphi_M(z^{-\infty}(t)))^T$ by

$$\varphi_i(z^{-\infty}(t)) := -\int_t^{-\infty} \bar{\lambda}_i^u(t)[\sum_{j=1}^M b_{ij} f_j(z_j^{-\infty}(u)) + \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) g_j(z_j^{-\infty}(v))dv + I_i]du - \sum_{j=1}^M \int_t^{-\infty} \bar{\lambda}_i(t) h_{ij}^{\dagger}(z_j^{-\infty}(u)) d\omega_j(u),$$

where $i \in \mathrm{M}$. Now we need three steps to complete our proof.

Step1: We will prove that $\varphi(z^{-\infty}(t))$ is continuous. Since

$$\begin{aligned} E|&\int_{t+\Delta t}^{-\infty} \bar{\lambda}_i^u(t+\Delta t)[\sum_{j=1}^M b_{ij} f_j(z_j^{-\infty}(u)) + \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) g_j(z_j^{-\infty}(v))dv + I_i]du \\ &- \int_t^{-\infty} \bar{\lambda}_i^u(t)[\sum_{j=1}^M b_{ij} f_j(z_j^{-\infty}(u)) + \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) g_j(z_j^{-\infty}(v))dv + I_i]du|^2 \\ &= \int_{t+\Delta t}^{-\infty} \bar{\lambda}_i^u(t+\Delta t)[\sum_{j=1}^M b_{ij} f_j(z_j^{-\infty}(u)) + \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) g_j(z_j^{-\infty}(v))dv + I_i]du \end{aligned}$$

Step 2: We will show the process $x^{-\infty}(t)$ is stochastically bounded. For any $N > 0$,

$$\begin{aligned} P\{|x_i^{-\infty}(t)| > N\} \le\, & P\{|\int_t^{-\infty} \bar{\lambda}_i^u(t)[\sum_{j=1}^M b_{ij} f_j(x_j^{-\infty}(u)) + \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) g_j(x_j^{-\infty}(v))dv + I_i]du > \frac{N}{M+1}\} \\ &+ \sum_{j=1}^M P\{|\int_t^{-\infty} \bar{\lambda}_i^u(t) h_{ij}^{\dagger}(x_j^{-\infty}(u)) d\omega_j(u)| > \frac{N}{M+1}\}, \end{aligned} \quad (2.4)$$

$i \in \mathrm{M}$.

It is sufficient to prove that every term in the right side of (2.4) is stochastically bounded.

Let $r \in \mathbb{N}$ and $p \in (0, \min_{i\in\mathrm{M}}\{1 + 2a_i\alpha_{ii}^{-2}\})$. First, we claim that

$$P\{|\int_t^{-\infty} [\bar{\lambda}_i^u(t)(\sum_{j=1}^M b_{ij} f_j(x_j^{-\infty}(u)) + \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) g_j(x_j^{-\infty}(v))dv + I_i]^r du| > N^r\} \le L_1 N^{-p}, \quad (2.5)$$

where $L_1 < +\infty$ and the integral is defined for all trajectories. In fact, by Lemma 2.2, we have

$$\begin{aligned} &P\{|\int_t^{-\infty} [\bar{\lambda}_i^u(t)(\sum_{j=1}^M b_{ij} f_j(x_j^{-\infty}(u)) + \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) g_j(x_j^{-\infty}(v))dv + I_i]^r du| > N^r\} \\ &\le P\{\bigcup_{n=0}^{+\infty} |\int_{t-T(n+1)}^{t-Tn} [\bar{\lambda}_i^u(t)(\sum_{j=1}^M b_{ij} f_j(x_j^{-\infty}(u)) + \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) g_j(x_j^{-\infty}(v))dv + I_i]^r du| > N^r 2^{-n}\} \\ &\le \sum_{n=0}^{+\infty} P\{\int_{t-T(n+1)}^{t-Tn} [\bar{\lambda}_i^u(t)(\sum_{j=1}^M b_{ij}^{\hbar} |f_j(x_j^{-\infty}(u))| + \sum_{j=1}^M \int_{-\infty}^u k_{ij}(u-v) |g_j(x_j^{-\infty}(v))| dv + I_i^{\hbar})]^r du| > N^r 2^{-n}\} \\ &\le \sum_{n=0}^{+\infty} P\{\int_{t-T(n+1)}^{t-Tn} [\bar{\lambda}_i^u(t)(\sum_{j=1}^M b_{ij}^{\hbar} B_{f_j} + \sum_{j=1}^M k_{ij}^* B_{g_j} + I_i^{\hbar}]^r du > N^r 2^{-n}\} \\ &\le \sum_{n=0}^{+\infty} L N^{-p} q^n \le \frac{L}{1-q} N^{-p}. \end{aligned}$$

Second, we claim that

$$P\{|\int_t^{-\infty} \bar{\lambda}_i^u(t) h_{ij}^{\dagger}(x_j^{-\infty}(u)) d\omega_j(u)| > N^r\} \le L_2 N^{-p}, \quad (2.6)$$

for $r \in \mathbb{N}$ and $L_2 < +\infty$. Let $\omega_j^t(u) := \omega_j(t-u) - \omega_j(u)$. Since

$$\int_t^{t-v} \bar{\lambda}_i^u(t) h_{ij}^{\dagger}(x_j^{-\infty}(u)) d\omega_j(u) = \int_0^v \bar{\lambda}_i^{t-u}(t) h_{ij}^{\dagger}(x_j^{-\infty}(t-u)) d\omega_j^t(u)$$

is a martingale. Apply Doob's maximal inequality, there exist a constant $c < +\infty$ such that

$$\begin{aligned} &P\{|\int_t^{-\infty} \bar{\lambda}_i^u(t) h_{ij}^{\dagger}(x_j^{-\infty}(u)) d\omega_j(u)| > N^r\} \\ &\le \lim_{V\to\infty} P\{\sup_{0<v<V} |\int_t^{t-v} \bar{\lambda}_i^u(t) h_{ij}^{\dagger}(x_j^{-\infty}(u)) d\omega_j(u)| > N^r\} \\ &\le \lim_{V\to\infty} N^{-2r} E[\sup_{0<v<V} |\int_t^{t-v} \bar{\lambda}_i^u(t) h_{ij}^{\dagger}(x_j^{-\infty}(u)) d\omega_j(u)|] \\ &\le \lim_{V\to\infty} N^{-2r} cE[\int_t^{t-v} [\bar{\lambda}_i^u(t) h_{ij}^{\dagger}(x_j^{-\infty}(u))]\ du] \\ &\le \lim_{V\to\infty} N^{-2r} cE[-\int_t^{-\infty} [\bar{\lambda}_i^u(t) h_{ij}^{\dagger 2}(x_j^{-\infty}(u))]\ du]. \end{aligned}$$

So it remains to show that

$$E[-\int_t^{-\infty}[\lambda_i^u(t)h_{ij}^{\dagger 2}(x_j^{-\infty}(u))]\ du]<+\infty$$

for $t\in\mathbb{R}$. Taking $r=2$ and $p=2+\delta(0<\delta<\min_{i\in M}\{2a_i\alpha_{ii}^2-1\})$ int the prood of (2.5), we have

$$E[-\int_t^{-\infty}[\lambda_i^u(t)h_{ij}^{\dagger 2}(x_j^{-\infty}(u))]\ du]$$

$$\le 1+\sum_{n=0}^{+\infty}2^{2(n+1)}P\{4^n\le -\int_t^{-\infty}[\lambda_i^u(t)h_{ij}^{\dagger}(x_j^{-\infty}(u))]\ du\le 4^{n+}\}$$

$$\le 1+\sum_{n=0}^{+\infty}2^{2(n+1)}P\{4^n\le -\int_t^{-\infty}[\lambda_i^u(t)h_{ij}^{\dagger}(x_j^{-\infty}(u))]\ du\}$$

$$\le 1+\sum_{n=0}^{+\infty}2^{2(n+1)}L_1 2^{-n(2+\delta)}=1+\frac{4L_1}{1-2^{-\delta}}.$$

Thus, (2.6) holds and we have proved that the solution is stochastically bounded. Together with (2.5) and (2.6), one gets that

$$P\{|x^{-\infty}(t)|>N\}\le L_3N^{-p}, \qquad (2.7)$$

where $L_3<+\infty$. Using (2.7) for $p=2+\delta(0<\delta<\min_{i\in M}\{2a_i\alpha_{ii}^2-1\})$, we get

$$\begin{aligned}E|x^{-\infty}(t)|^2 &\le 1+\sum_{n=0}^{+\infty}2^{2(n+1)}P\{2^n\le|x^{-\infty}(t)|\le 2^{n+1}\}\\ &\le 1+\sum_{n=0}^{+\infty}2^{2(n+1)}P\{2^n\le|x^{-\infty}(t)|\}\\ &\le 1+\sum_{n=0}^{+\infty}2^{2(n+1)}L_3 2^{-n(2+\delta)}\\ &=1+\frac{4L_3}{1-2^{-\delta}}<+\infty\end{aligned} \qquad (2.8)$$

Step 2: Let $B_E^{\infty}(\mathbb{R},L^2(P,\mathbb{R}^M))$ be the collection of all stochastic bounded process $x:\mathbb{R}\to L^2(P,\mathbb{R}^M)$ with $E|x^{-\infty}(t)|^p\le\frac{4L_3}{1-2^{-\delta}}$. Obviously, $B_E^{\infty}(\mathbb{R},L^2(P,\mathbb{R}^M))\subset B_E(\mathbb{R},L^2(P,\mathbb{R}^M))$ is a Banach space. Define $\varphi(x^{-\infty}(t)):=(\varphi_1(x^{-\infty}(t)),\cdots\varphi_M(x^{-\infty}(t)))^T$ by

$$\begin{aligned}\varphi_i(x^{-\infty}(t)) &:= -\int_t^{-\infty}\lambda_i^u(t)[\sum_{j=1}^M b_{ij}f_j(x_j^{-\infty}(u))\\ &+\sum_{j=1}^M\int_{-\infty}^u k_{ij}(u-v)g_j(x_j^{-\infty}(v))dv+I_i]du\\ &-\sum_{j=1}^M\int_t^{-\infty}\lambda_i^u(t)h_{ij}^{\dagger}(x_j^{-\infty}(u))d\omega_j(u),\end{aligned}$$

where $i\in M$ and $x^{-\infty}(t)\in B_E^{\infty}(\mathbb{R},L^2(P,\mathbb{R}^M))$. It follows from Step 1 that φ maps $B_E^{\infty}(\mathbb{R},L^2(P,\mathbb{R}^M))$ into itself. To complete the proof, we will prove that φ has a unique fix-point. For any $x^{-\infty}(t),y^{-\infty}(t)\in B_E^{\infty}(\mathbb{R},L^2(P,\mathbb{R}^M))$, we get

$$\begin{aligned}&\varphi_i(x^{-\infty}(t))-\varphi_i(y^{-\infty}(t))=\\ &-\int_t^{-\infty}\lambda_i^u(t)\sum_{j=1}^M b_{ij}[f_j(x_j^{-\infty}(u))-(y_j^{-\infty}(u))]du\\ &+\sum_{j=1}^M\int_{-\infty}^u k_{ij}(u-v)[g_j(x_j^{-\infty}(v))-g_j(y_j^{-\infty}(v))]dv]du\\ &-\sum_{j=1}^M\int_t^{-\infty}\lambda_i^u(t)[h_{ij}^{\dagger\dagger}(x_j^{-\infty}(u))-h_{ij}(y_j^{-\infty}(u))]d\omega_j(u),\quad i\in M\end{aligned}$$

It is obviously that

$$\begin{aligned}&\varphi_i(x^{-\infty}(t))-\varphi_i(y^{-\infty}(t))\\ &\le\sum_{j=1}^M b_{ij}^{\hbar}\int_t^{-\infty}\lambda_i^u(t)|f_j(x_j^{-\infty}(u))-f_j(y_j^{-\infty}(u))|du\\ &+\sum_{j=1}^M\int_{-\infty}^u k_{ij}(u-v)[g_j(x_j^{-\infty}(v))-g_j(y_j^{-\infty}(v))]dv]du\\ &+\sum_{j=1}^M\int_t^{-\infty}\lambda_i^u(t)[h_{ij}^{\dagger\dagger}(x_j^{-\infty}(u))-h_{ij}(y_j^{-\infty}(u))]d\omega_j(u),\quad i\in M\end{aligned}$$

Since $(\sum_{i=1}^{n_1}r_i)^2\le\sum_{i=1}^{n_1}n_1r_i^2$, we can write:

$$\begin{aligned}&\mathrm{E}|\varphi_i(x^{-\infty}(t))-\varphi_i(y^{-\infty}(t))|^2\\ &\le E[\sum_{j=1}^M 3M(L_{f_j}b_{ij}^{\hbar})^2[\int_t^{-\infty}\lambda_i^u(t)|x_j^{-\infty}(u)-y_j^{-\infty}(u)|du]^2\\ &+\sum_{j=1}^M 3M(L_{g_j})^2[\int_t^{-\infty}\lambda_i^u(t)[\int_{-\infty}^u k_{ij}(u-v)|x_j^{-\infty}(v)-y_j^{-\infty}(v)|dv]du]^2\\ &+\sum_{j=1}^M 3M(L_{h_{ij}^{\dagger}})^2[\int_t^{-\infty}\lambda_i^u(t)(x_j^{-\infty}(u)-y_j^{-\infty}(u))d\omega_j(u)]^2]\qquad i\in M.\end{aligned}$$

Let $\pi_i^s(t)=\exp\{a_i^*(t-s)+2\alpha_{ii}[\omega_i(t)-\omega_i(s)]\}$. Then, $E(\pi_i^s(t))^p:=\exp\{[a_i^*+2p\alpha_{ii}](t-s)p\}$ for any $p\in\mathbb{R}$. Now, using Cauchy-Swhwarz inequality we can write:

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$$
\begin{aligned}
&\mathrm{E}|\varphi_i(x^{-\infty}(t))-\varphi_i(y^{-\infty}(t))|^2\\
&\le E[\sum_{j=1}^{M}3M(L_{f_j}b_{ij}^{\hbar})^2[\int_t^{-\infty}\exp\{a_i^*(t-u)\}du][\int_t^{-\infty}\lambda_i^u(t)\,|\,x_j^{-\infty}(u)-y_j^{-\infty}(u)\,|\,du]^2\\
&+\sum_{j=1}^{M}3M(L_{g_j})^2[\int_t^{-\infty}\exp\{a_i^*(t-u)\}du][\int_t^{-\infty}\lambda_i^u(t)[\int_{-\infty}^{u}k_{ij}(u-v)\,|x_j^{-\infty}(v)-y_j^{-\infty}(v)\,|\,dv]du]^2\\
&+\sum_{j=1}^{M}3M(L_{h_{ij}^{\dagger}})^2[\int_t^{-\infty}\lambda_i^u(t)(x_j^{-\infty}(u)-y_j^{-\infty}(u))d\omega_j(u)]^2]\\
&\le\sum_{j=1}^{M}3M\frac{(L_{f_j}b_{ij}^{\hbar})^2}{a_i^*}\int_t^{-\infty}E\pi_i^u(t)du\sup_{t\in\mathbb{R}}E\,|\,x_j^{-\infty}(u)-y_j^{-\infty}(u)\,|^2\\
&+\sum_{j=1}^{M}3M\frac{(L_{g_j})^2}{a_i^*}[\int_t^{-\infty}E\pi_i^u(t)[\int_{-\infty}^{u}k_{ij}(u-v)dv]^2du]\sup_{t\in\mathbb{R}}E\,|\,x_j^{-\infty}(u)-y_j^{-\infty}(u)\,|^2\\
&+\sum_{j=1}^{M}3M(L_{h_{ij}^{\dagger}})^2[\int_t^{-\infty}E\pi_i^u(t)^2du]\sup_{t\in\mathbb{R}}E\,|\,x_j^{-\infty}(u)-y_j^{-\infty}(u)\,|^2\,.
\end{aligned}
$$

It follows from (2.1) that

$$
\begin{aligned}
&\mathrm{E}|\varphi_i(x^{-\infty}(t))-\varphi_i(y^{-\infty}(t))|^2\\
&\le\sum_{j=1}^{M}3M\frac{(L_{f_j}b_{ij}^{\hbar})^2}{a_i^*a_{i1}^{\dagger}}\sup_{t\in\mathbb{R}}E\,|\,x_j^{-\infty}(u)-y_j^{-\infty}(u)\,|^2\\
&+\sum_{j=1}^{M}3M\frac{(L_{g_j})^2}{a_i^*a_{i1}^{\dagger}}(\int_0^{+\infty}k_{ij}(v)dv)^2\sup_{t\in\mathbb{R}}E\,|\,x_j^{-\infty}(u)-y_j^{-\infty}(u)\,|^2\\
&+\sum_{j=1}^{M}3M\frac{(L_{h_{ij}^{\dagger}})^2}{a_{i2}^{\dagger}}\sup_{t\in\mathbb{R}}E\,|\,x_j^{-\infty}(u)-y_j^{-\infty}(u)\,|^2\\
&\le 3M\sum_{j=1}^{M}[\frac{(L_{f_j}b_{ij}^{\hbar})^2}{a_i^*a_{i1}^{\dagger*\dagger\dagger}}+\frac{(L_{g_j})^2}{a_i\,a_{i1}}+\frac{(L_{h_{ij}^{\dagger}})^2}{a_{i2}}]\sup_{t\in\mathbb{R}}E\,\|\,x_j^{-\infty}(u)-y_j^{-\infty}(u)\,\|^2
\end{aligned}
$$

Thus, it follows that

$$
\begin{aligned}
&\mathrm{E}\|\varphi_i(x^{-\infty}(t))-\varphi_i(y^{-\infty}(t))\|^2\\
&\le\max_{i\in\mathrm{M}}\{3M\sum_{j=1}^{M}[\frac{(L_{f_j}b_{ij}^{\hbar})^2}{a_i^*a_{i1}^{*\dagger\dagger\dagger}}+\frac{(L_{g_j})^2}{a_i\,a_{i1}}+\frac{(L_{h_{ij}^{\dagger}})^2}{a_{i2}}]\}\\
&\times\sup_{t\in\mathbb{R}}E\,\|\,x_j^{-\infty}(u)-y_j^{-\infty}(u)\,\|^2\\
&\le\Theta\sup_{t\in\mathbb{R}}E\,\|\,x_j^{-\infty}(u)-y_j^{-\infty}(u)\,\|^2\,.
\end{aligned}
$$

Consequently, if $\Theta<1$, then (1.1) has a unique fixed-point in $B_E^{\infty}(\mathbb{R},L^2(P,\mathbb{R}^M))$, which can be express explicitly by (2.3). The proof is complete.

3 Conclusions

By using stochastic integral properties of homogeneous linear equations and fixed-point theorem, we investigate bounded dynamics of delayed neural networks with stochastic effects. Some new criteria for the existence of a unique stochastically bounded solution of stochastic networks are given. Our results can be generalized to nonautonomous cases.

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